$$\therefore \frac{\Delta R}{R} = -\frac{a\cos\pi}{\sin^2\pi}\cdot\frac{\sin\pi}{a}\,\Delta\pi \times \text{circ. meas. of } 1'' = -\frac{\cos\pi}{\pi}\cdot\Delta\pi,$$

$$\therefore PP' = -\Delta R = \frac{R\cos\pi}{\pi}\cdot\Delta\pi, \text{ and } Q = \theta,$$

$$\therefore P'R = \frac{R\cos\pi.\Delta\pi}{\pi}\cdot\frac{\cos(N+\psi)\cos\theta+\sin(N+\psi)\sin\theta}{\cos\psi}$$

$$= \frac{\cos\pi.\Delta\pi}{\pi}\cdot\frac{y\cos(N+\psi)+x\sin(N+\psi)}{\cos\psi},$$

where x and y are the co-ordinates of the Moon's centre at the time T.

Since these small changes are independent of one another, their combined effect will be equal to their sum.

$$\therefore \left.\begin{matrix}\text{total correction to the}\\ \text{time T in seconds}\end{matrix}\right\} = \frac{\text{sum of the several values of P'R}}{n}\times h,$$

where h is the number of seconds in the unit of time; that is

$$\text{correction} = -\frac{h}{n\pi\cos\psi}[\sin(N+\psi)\cos\delta.\Delta(\alpha-\alpha')+\cos(N+\psi)\Delta(\delta-\delta')$$
$$+\pi.\Delta\kappa-\{x\sin(N+\psi)+y\cos(N+\psi)\}\cos\pi.\Delta\pi],$$

which is the usual formula, omitting the terms depending on error of eccentricity of the meridian.

1876, *July*.

On the Effect of Wear in the Micrometer-Screws of the Greenwich Transit-Circle. By W. H. M. Christie, Esq.

It has been the practice of observers to examine carefully the accuracy of micrometer-screws on receiving them from the maker's hands; but no one, so far as I am aware, has considered it necessary to make any observation afterwards, though the effect of wear, when observations are made frequently, soon becomes sensible. Under these circumstances, it may be desirable to give an account of a case in which considerable errors have been caused by continued use during a period of more than twenty years.

The wear in the micrometer-screws of the Greenwich Transit-Circle first showed itself by a discordance between the Zenith-points deduced from the Nadir observation and from Stars respectively; though the cause was not suspected till the beginning of 1875, when a change in the position of the division in the field of view of the micrometer-microscopes was found to be accompanied by a change in the discordance from $+0''\cdot7$ to $-0''\cdot6$. This discordance first became sensible in the year 1868; but the error in the

screws, as will be seen presently, existed certainly before that time, though its effect on the Nadir observation was insensible so long as the observations were distributed over the whole range of the screws. This was the case up to 1868, after which the system was altered, the Nadir observation being from that time made with the circle-divisions at 0^r approximately. When the change in the discordance occurred at the beginning of 1875, the values of successive revolutions of the telescope-micrometer, and also of the microscope-micrometer-screws were tested by means of the collimator, and it was found that there was considerable inequality in the latter. To obtain the corrections applicable to the different parts of the screws, four supplementary microscopes (which had only been used occasionally for determining the division-errors of the circle) were mounted and read for comparison with the six ordinary microscopes. These observations are printed in the *Greenwich Observations*, 1875. From 457 comparisons thus made, the continuous curve shown in the diagram has been laid down, the means of the separate apparent corrections tabulated for every tenth of a revolution having been smoothed by taking the means for each adjacent pair, and then the means of these results again in the same way. The ordinates give the corrections to the circle-reading corresponding to the revolutions and tenths of Microscope A, indicated above, which has been taken for convenience of application. Its readings were throughout $0^r.13$ less than the mean of the six micrometer-readings.

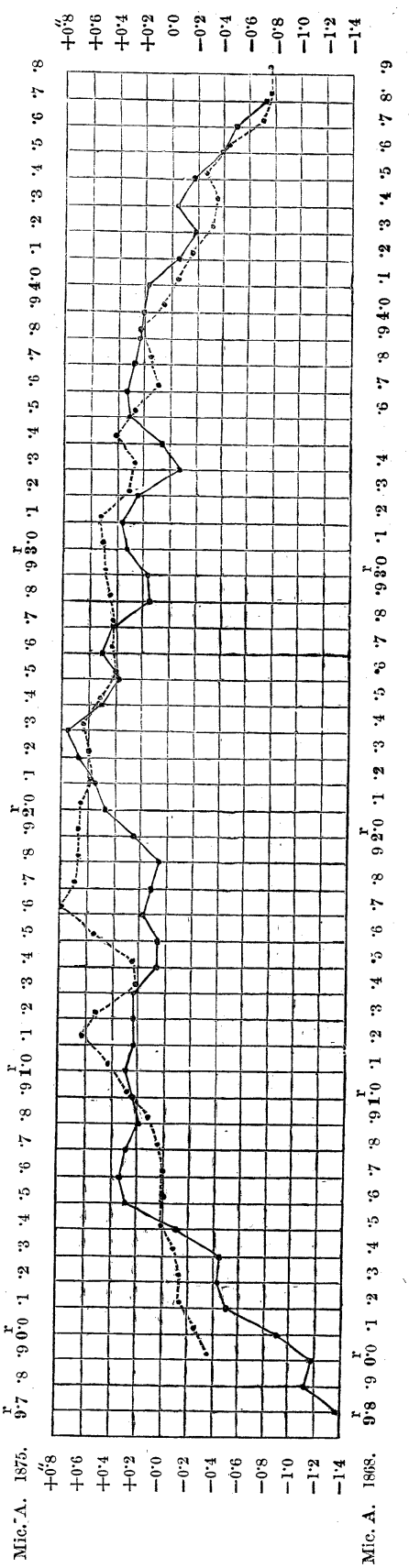
In 1868 similar observations had been made, though with a different object in view, namely, to determine the probable value of the outstanding division-error of the circle. They are, however, equally available for testing the revolutions of the micrometer-screws, and from 289 comparisons thus made, the corrections applicable in 1868 have been found. These are represented by the dotted curve, formed in the same way as that for 1875, but with the origin shifted $0^r.08$ to the left, since Microscope A in 1868 read on the average $0^r.05$ less than the mean. The revolutions and tenths of Microscope A for the 1868 curve are given below. Both curves are thus virtually referred to the mean of the micrometer-readings. It will be remarked that the two curves are generally similar, giving large positive corrections for the middle of the range; but that the curve for 1868 falls above that for 1875, the corrections being on the whole positive. In fact, the mean of all the individual observations of discordance in 1868 is $+0''.20$, whilst in 1875 it is $+0''.09$. As these quantities considerably exceed the probable error of the result, which would be about $0''.03$, it is possible that some systematic error, such as a difference in the outstanding division-error for the parts of the circle used in finding the Zenith-point with the ordinary and supplementary microscopes respectively, may have affected all the observations. This, however, does not interfere with the comparison of the

two curves, as the ordinates may be supposed reduced throughout by + 0″·20 and + 0″·09 in the two cases respectively. There is another point of difference between the two curves, which depends on the adopted correction for runs, and has the effect of making the axis of symmetry fall somewhat to the left in the dotted curve, which consequently lies above the other at 0ʳ, and falls below it at 5ʳ. Now the wear will take place on the part of the thread against which the spring presses the hollow screw of the micrometer slide, and the cross-wires will consequently be carried further from the head, that is in the direction of increasing readings. Thus, for any given position of the cross-wires, the head will give too small a reading, and a positive correction will therefore be required, this correction being larger for the middle of the range of the screw, which is the part most frequently used. One effect of this will be to increase the run, if the error at the large reading (4ʳ·9 approx.) be greater, algebraically, than that at the small reading (0ʳ approx.), and the same will apply to the correction for runs if corrections be substituted for errors. Now the corrections increase (algebraically) from negative to small positive readings, and decrease from large positive readings onwards, and thus the correction for runs will be greater (algebraically) when the small reading is taken on the negative side of the zero than when it falls on the positive side. In the former case the corrections for error in the screws will be algebraically increased for small readings and decreased for large readings by application of the correction for runs; and this effect is purely arbitrary, since we may choose any part of the screw for zero.

Making due allowance for this, and also for the difference in the sums of all the corrections for the two sets, the dotted curve must be lowered 0″·4 approximately at 0ʳ, and raised 0″·2 at 5ʳ, proportional changes being made for intermediate readings. The agreement between the two curves will then be remarkably close, except for the portion between 0ʳ and 1ʳ, at which point the 1875 curve rises very abruptly, indicating that considerable wear has taken place in that part since 1868. This appears to be fully accounted for by the circumstance that in that year the practice was commenced of taking the Nadir observation with the circle-division at 0ʳ approximately; and, as the observation is regularly made twice a day, there would be a preponderance of some 4,000 observations at this part of the screw in the course of the eight years elapsed since 1868.

It would thus appear that by far the greater portion of the wear in the micrometer-screws has taken place in the seventeen years preceding 1868, and that, except in the part between 0ʳ and 1ʳ, the errors have not sensibly changed since that date. On comparing the several tenths of any one revolution, the corrections for ·7, ·8, and ·9 would appear to be about 0″·2 smaller (algebraically) than the others, but the difficulty of eliminating the effect of the larger corrections is considerable, and the quantities are too small to be determined with certainty.

Corrections to Micrometer-Screws of Greenwich Transit-Circle, 1868 and 1875.



The corrections thus found are considerable; and it is to be remarked that they are of the nature of systematic errors, affecting all the observations of any one star, which would usually be observed in the same part of the field of view of the telescope, and therefore, for several years, with the circle-division about the same part of the range of the micrometer-screw. This points to the necessity for periodic examination of micrometer-screws, though the effect of wear will be sensibly eliminated if the action of the spring be in opposite directions for every pair of micrometers, the readings increasing towards the head for one micrometer, and from the head for the other of each pair.

It may be proper to add, that the accuracy of the screws of the four supplementary microscopes has been tested indirectly by comparison with new screws applied to the six ordinary microscopes, as well as directly by observations on the collimator.

Royal Observatory, Greenwich,
1876, *Nov.* 9.

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*Spectroscopic Results for the Motions of Stars, and of Venus in the Line of Sight, and for the Rotation of the Sun and of Jupiter, made at the Royal Observatory, Greenwich. II.**

(*Communicated by the Astronomer Royal.*)

The recent results for Star-motions have been obtained in the same manner as those given in the *Monthly Notices* for last May, and the general arrangement is unaltered. As, however, it has been suggested by P. Secchi that the results may possibly be dependent on the position of the spectroscope, the reading of the position-circle (or the position-angle of the slit) is here added. The slit lies north and south, and the deviation of the ray is towards the east when the reading is 0°. This is the most convenient position, and is that in which most of the observations have been made. In the earlier measures made in 1874 observations were taken in each of the four quadrants; but, as the position of the spectroscope did not appear to affect the result, the practice was discontinued. All the observations since then are therefore strictly comparable in this respect, and all stars would be equally affected by any error depending on the position of the spectroscope. As a check on the absolute values, however, some measures of the lines in the spectrum of the Moon and of *Venus* have been made in different positions. Special care has been taken to observe the stars in pairs east and west as far as practicable, so that the Earth's motion might affect the several observations with different signs, and also that any error depending on the position of the instrument might be eliminated.

The means have been taken for the whole series of observations of each star, including those given in the *Monthly Notices*

* The Results for the Rotation of the Sun and of *Jupiter* will be given in the *December* number.—[Ed.]